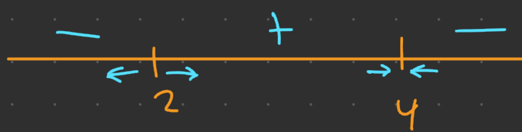
Question 5

Fall 2024 Exam 1

Consider a population of fish that is regularly harvested and has population $x(t)$ which is modeled by the "Logistic-with-harvest" equation $x' = x(6-x) - 8$. Which of the following is true about the critical points of the system?

$$x' = x(6-x) - 8$$

$$x' = 6x - x^2 - 8$$
$$= -(x^2 - 6x + 8)$$
$$x' = -(x-4)(x-2)$$



Question 1

Fall 2024 Final

A population P(t) is modeled by the logistic equation

$$P' = \frac{1}{12}P(12 - P)$$

and initial value $P(0) = 3$. What will the population P(t) be when $t = \ln 3$?

$$P' = P\left(\underset{\substack{\uparrow \\ r}}{1} - \frac{P}{\underset{\substack{\uparrow \\ K}}{12}}\right)$$

$$P = \frac{K}{1 + Ce^{-rt}}$$

$$P = \frac{12}{1+Ce^{-t}}$$

$$3 = \frac{12}{1+Ce^{0}} = \frac{12}{1+C}$$

$$3(1+C) = 12$$
$$1+C = 4$$
$$C = 3$$

$$P(t) = \frac{12}{1+3e^{-t}}$$

$$P(\ln 3) = \frac{12}{1+3e^{-\ln 3}}$$

$$e^{-\ln 3} = e^{\ln 1/3} = 1/3$$

$$= \frac{12}{1+3(1/3)} = \frac{12}{2} = 6$$

Question 9

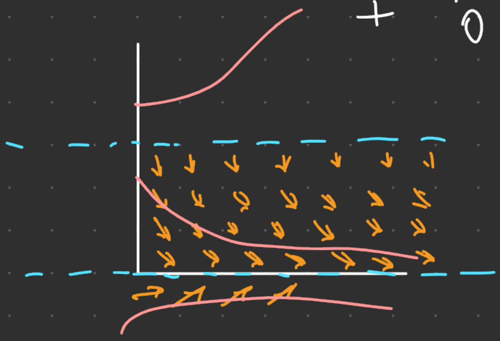
Fall 2023 Exam 1

A population's size at time t is $x(t)$ and is modeled by the differential equation

$$\frac{dx}{dt} = \frac{1}{3}x(x-3).$$

(a) Sketch a phase diagram for the differential equation. What does this phase diagram tell you about $\lim_{t\to\infty} x(t)$ if $x(0) = 1$?

(b) Find an explicit solution to the differential equation when $x(0) = 1$.



$$x = \frac{K}{1+e^{-rt}}$$

$$= \frac{3}{1+Ce^{t}}$$

$$x(0) = 1$$

$$x(0) = 1 = \frac{3}{1+Ce^{0}} = \frac{3}{1+C}$$

$$1+C = 3 \Rightarrow C = 2$$

$$x(t) = \frac{3}{1+2e^{t}}$$